Semidefinite Programming Duality: Implications for System Theory and Computation

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Abstract

Several important problems in systems and control theory can be reformulated as semidefinite programming (SDP) problems, i.e., minimization of a linear objective subject to Linear Matrix Inequality (LMI) constraints. From convex optimization duality theory, conditions for infeasibility of the LMIs as well as dual optimization problems can be formulated. These can in turn be re-interpreted in control or system theoretic terms, often yielding new results or new proofs for existing results from control theory. We present such connections for a few problems associated with linear time-invariant systems.

We then turn to computational issues. The reduction of a control problem to an SDP problem often requires a large number of auxiliary variables, so that the resulting SDP problem can be very large, even though the underlying control problem is not particularly large-scale. This limits the problem sizes that can be handled by general-purpose SDP solvers, and hence the applicability of SDP in practical engineering problems. This is true for example with SDP problems where the underlying LMIs have a special form that is typically encountered with the application of the Kalman-Yakubovich-Popov (KYP) Lemma. We discuss efficient implementations of primal-dual interior-point methods for SDP problems with KYP LMIs, and show how orders-of-magnitude savings in computation can be realized when problem structure is exploited using straightforward linear algebra techniques.

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References

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