Generalized interpolation in H^∞ with applications to systems and control and signal processing

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Abstract. In a well-known paper [1], Sarason generalized some classical interpolation problems for H^∞ functions on the unit disc with norm not greater than one to problems concerning operators on a coinvariant subspace $\mathcal{K} = H^2 \ominus \phi H^2$, where ϕ is an inner function. These operators have norm not greater than one, and, among his results, he studied the structure of generalized interpolants for operators having norm one. In a variety of interesting cases, there is a unique such interpolant, which is given by the quotient of functions in $\mathcal{K}$. Following [2], in this talk we consider the case where the operator is a strict contraction. There turns out to be an infinite number of interpolants that are such quotients, and we give a complete parameterization of these. Each such interpolant can be obtained as the unique minimizer to a strictly convex functional.

Our methodology follows that of [3, 4] (also see references therein) and is inspired by the engineering applications of classical interpolation problems in circuits, systems and signal processing, cases which all deal with the situation where ϕ is a finite Blaschke product and in which the quotient representation is physically natural. We give several examples of such applications. We also demonstrate that the case when ϕ is a singular inner function has a natural systems-theoretical interpretation.

References

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