Convolutional Codes, Systems over Finite Fields and Fault Tolerance

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Abstract

It is well known that a convolutional code is essentially a linear system defined over a finite field. Despite this well known connection convolutional codes have been studied in the past mainly by graph theoretic methods and in contrast to the situation of block codes there exist only few algebraic constructions. It is a fundamental problem in coding theory to construct convolutional codes with a designed distance.

A first part of the talk describes the connection between convolutional codes and linear systems [3, 4]. Using systems theoretic methods we explain how to construct codes with maximal or near maximal free distance [5]. We show how decoding can be viewed as a discrete tracking problem where the received signals have to be optimally matched with a sequence generated by the encoder. We also report on recent progress in the construction of convolutional codes by algebraic means [1].

Convolutional codes have been used in the past mainly for the purpose of point to point communication. Recent work by Hadjicostis, Verghese, Fliess and their collaborators have shown (see e.g. [2]) interesting applications to Fault Tolerant systems where codes over a large alphabet play an important role. In a final part of the talk we will address these applications.

References

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