Model reduction, realization and Hankel theory for nonlinear systems

Jacquelien M.A. Scherpen
Delft Center for Systems and Control, Delft University of Technology
Mekelweg 2, 2628 CD Delft, The Netherlands
http://www.dcsc.tudelft.nl/~jscherpen , J.M.A.Scherpen@dcsc.tudelft.nl

In the theory of continuous time linear systems, the system Hankel operator plays an important role in a number of problems. For example, when viewed as mapping from past inputs to future outputs, it plays a direct role in the abstract definition of state. It also plays a central role in minimality theory, in model reduction problems, in realization theory, and related to these, in linear identification methods. Specifically, the Hankel operator supplies a set of similarity invariants, the so called Hankel singular values, which can be used to quantify the importance of each state in the corresponding input-output system. The Hankel operator can also be factored into the composition of an observability and controllability operator, from which Gramian matrices can be defined and the notion of a balanced realization follows, first introduced in [2], and further studied by many authors, e.g., [5]. The Hankel singular values are most easily computed in a state-space setting using the product of the Gramian matrices, though intrinsically they depend only on the given input-output mapping. The linear Hankel theory is rather complete and the relations between and interpretations in the state-space and input-output settings are fully understood.

The continuous time nonlinear extension of the linear theory is the topic of this presentation. A brief overview will be given about previous results on the controllability and observability function and their relation with minimality, e.g., [3]. Furthermore, the relation with the Hankel operator will be considered and a combination of state-space and input-output thinking results in the so-called axis-singular value functions that correspond to an input-normal/output-diagonal representation, e.g., [1]. In these developments we rely heavily on the concept of a nonlinear Hilbert adjoint as presented in [4]. This is a main ingredient for the Hankel singular value analysis that characterizes the axis-singular value functions. Based on this, a nonlinear state-space system is brought in balanced form. This state-space realization forms a basis for a new balanced model reduction scheme for nonlinear systems.

References

[1] K. Fujimoto, J.M.A. Scherpen, Nonlinear input-normal realizations based on the differential eigenstructure of Hankel operators, to appear in *IEEE Trans. Automatic Control*.

[2] B. C. Moore, Principal component analysis in linear systems: controllability, observability and model reduction, *IEEE Trans. Automatic Control*, AC-26:17–32, 1981.

[3] J.M.A. Scherpen, W.S. Gray, Minimality and Local State Decompositions of a Nonlinear State Space Realization using Energy Functions, *IEEE Trans. Automatic Control*, AC-45, 11 (2000) 2079-2086

[4] J.M.A. Scherpen, W.S. Gray, Nonlinear Hilbert Adjoints: Properties and Applications to Hankel Singular Value Analysis, *Nonlinear Analysis, Theory, Methods and Applications*, Vol. 51, 5 (2002) 883-901.

[5] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*, Prentice-Hall, Inc., Upper Saddle River, N.J., 1996.