Controllability For Systems Of Partial Differential Equations

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Abstract

In this talk I explain the *behavioural* generalisation of Kalman's fundamental notion of state space controllability [1] due to Willems [4], and beyond to distributed systems [2]. It turns out that controllability is now identical to the notion of a *potential* in physics or *vanishing homology* in mathematics.

This is the 'external side of it'. The larger purpose of the talk is to explain and justify a modelling process and the resulting class of models. A model is a picture of reality, and the closer it is to reality, the better the picture it will be and the more the nail will have been hit on the head. The closest we can get to reality, as we imagine it, is to take together every variation (with respect to space, time, et cetera) of the attributes of the system, the collection of all trajectories as it were, itself as the model. This collection considered all together in ones mind as an object of study is the *behaviour* of the system and is completely analogous to Poincare's conception of the collection of all phase curves of a vector field - the phase portrait - as an object of study. A priori perhaps any variation of the attributes of the system could have occured, but the *laws* of the system which these variations must obey specify those that actually do.

I consider here linear shift invariant behaviours specified by *local* laws, that is by partial differential equations. I justify this behavioural approach by the remarkable fact that controllability now is nothing more, nor less, than the existence of a potential. Equally important is the fact that the behavioural theory is computationally effective, for instance controllability of a distributed system can be determined by a *finite* procedure.

This entire development relies and builds on classical work of Hörmander, Malgrange, Palamodov and others in partial differential equations and several complex variables. It also requires the setting up of PDE equivalents of fundamental notions from algebraic geometry - the Hilbert Nullstellensatz, the radical of an ideal, complete variety and the elimination problem. The behavioural theory thus lies in the intersection of many important streams, from physics, mathematics and engineering [3].

References

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