

## Outer factorizations in one and several variables

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The Fejer-Riesz lemma for trigonometric polynomials states that  $q(z) \geq 0$ ,  $|z| = 1$ , if and only if there exists a polynomial  $p(z)$  so that  $q(z) = |p(z)|^2$ ,  $|z| = 1$ . In addition, one may choose  $p$  to be void of roots inside the open unit circle (i.e.,  $p$  is *outer*). Though simple to state and prove, the lemma has many useful applications, e.g., in filter design,  $H_\infty$  control, and wavelet theory. The result has been generalized to matrix valued and operator valued single variable trigonometric polynomials.

In this talk we explore multivariable generalizations of the Fejer-Riesz lemma. As is well-known, mere positivity of a trigonometric polynomial is not sufficient for the existence of a sums of moduli squared representation, let alone for a factorization as a single modulus squared. This observation, which goes back to Hilbert, has led to active research in sums of squares problems, multivariable factorization, and related multivariable interpolation problems. In this talk we will raise several questions, and answer some. The talk is based on joint papers with J. S. Geronimo, M. A. Dritschel, and Y. Hachez.

### REFERENCES

1. Michael A. Dritschel and Hugo J. Woerdeman, *Outer factorizations in one and several variables*, preprint.
2. Jeffrey S. Geronimo and Hugo J. Woerdeman, *Positive extensions, Riesz-Fejér factorization and autoregressive filters in two variables*, *Annals of Mathematics*, to appear.
3. Yvan Hachez and Hugo J. Woerdeman, *Approximating sums of squares with a single square*, preprint.